W_3 Minimal Models

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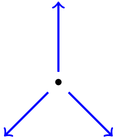
Statistical mechanics

2-state Potts model (Ising)

- Undergoes a phase transition
- Virasoro minimal model

3-state Potts model

- Virasoro minimal model
- Larger symmetry algebra than Virasoro[Zamolodchikov, '85]
- W_3 minimal model



Minimal Models

The **Virasoro minimal models** $M(p,q)$ are parametrised by $p,q \in \mathbb{Z}_{\geq 2}, (p,q) = 1$. The corresponding central charge is

$$c = 1 - \frac{6(p-q)^2}{pq}$$

The W_3 **minimal models** $WM(p,q)$ are parametrised by $p,q \in \mathbb{Z}_{\geq 3}, (p,q) = 1$. The corresponding central charge is

$$c = 2 - \frac{24(p-q)^2}{pq}$$

We are interested in the irreducible modules of these vertex operator algebras.

Irreducible modules of $M(p, q)$

Irreducible modules of the Virasoro minimal models[Wang, '93] are irreducible highest weight modules of the Virasoro algebra

There is a correspondence[Ridout,Wood, '15] between

Irreducible modules $\longleftrightarrow$ Zeros of the $\mathfrak{sl}(2)$ Selberg Integral

where the $\mathfrak{sl}(2)$ Selberg integral is defined as

$$S_n(\alpha, \beta, \gamma) = \oint_\Gamma \prod_{i=1}^{n} t_i^{\alpha-1}(1-t_i)^{\beta-1} \prod_{1\leq i<j\leq n} (t_i - t_j)^{2\gamma} \mathrm{d}t_1 \ldots \mathrm{d}t_n$$

M(p,q) Irreps to Selberg integral

Virasoro minimal model$= \frac{V}{I}$

$\downarrow$

If $V \curvearrowright M$, then $\frac{V}{I} \curvearrowright M \Longleftrightarrow I \curvearrowright M = 0$

$\downarrow$

Get constraint from $v \curvearrowright M = 0$, $\langle v \rangle = I$

$\downarrow$

zeros of $\mathfrak{sl}(2)$ Selberg

Want to generalise this idea to $WM(p,q)$

$WM(p,q)$ Irreps to Selberg integral

For the W_3 minimal model, one of the constraint is a $\mathfrak{sl}(3)$ Selberg integral

$$S(\beta_1,\beta_2) = \int_\Delta \prod_{1\le i<j\le n} (t_i - t_j)^{2\gamma}(s_i - s_j)^{2\gamma} \prod_{i=1}^{n}\prod_{j=1}^{n}(t_i - s_j)^{-\gamma}$$

$$\times \prod_{i=1}^{n} t_i^{-n\gamma}(1-t_i)^{\beta_1} \prod_{i=1}^{n} s_i^{-n\gamma}(1-s_i)^{\beta_2}$$

$$\times \prod_{i=1}^{n} \mathrm{d}x_i \prod_{i=1}^{n} \mathrm{d}y_i$$

$\mathfrak{sl}(3)$ Selberg integrals in the Literature

- [Tarasov, Varchenko, '03] ($\beta_1 = 1$ or $\beta_2 = 1$)

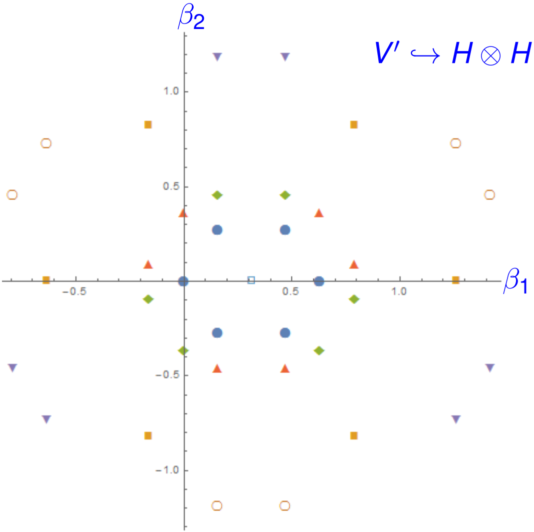
$$\int_{\Delta_1} \prod_{1\le i<j\le n_1} (t_i - t_j)^{2\gamma} \prod_{1\le i<j\le n_2} (s_i - s_j)^{2\gamma} \prod_{i=1}^{n_1}\prod_{j=1}^{n_2}(t_i - s_j)^{-\gamma} \times \prod_{i=1}^{n_1} t_i^{\alpha_1 -1} \prod_{i=1}^{n_2} s_i^{\alpha_2 -1} (1-s_i)^{\beta_2 -1} \prod_{i=1}^{n_1} \mathrm{d}x_i \prod_{i=1}^{n_2} \mathrm{d}y_i$$

- [Warnaar, '10] ($\beta_1 + \beta_2 = \gamma + 1$)

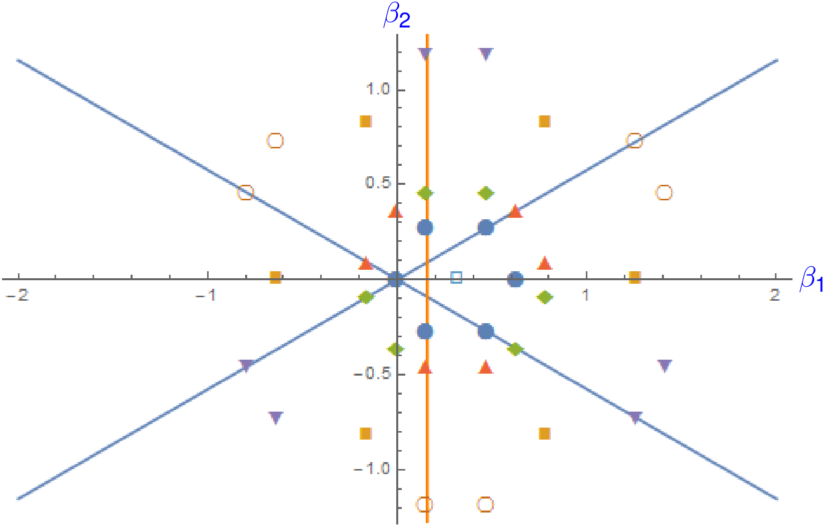
$$\int_{\Delta_2} \prod_{1\le i<j\le n_1} (t_i - t_j)^{2\gamma} \prod_{1\le i<j\le n_2} (s_i - s_j)^{2\gamma} \prod_{i=1}^{n_1}\prod_{j=1}^{n_2}(t_i - s_j)^{-\gamma} \times \prod_{i=1}^{n_1} t_i^{\alpha_1 -1} (1-t_i)^{\beta_1 -1} \prod_{i=1}^{n_2} s_i^{\alpha_2 -1} (1-s_i)^{\beta_2 -1} \prod_{i=1}^{n_1} \mathrm{d}x_i \prod_{i=1}^{n_2} \mathrm{d}y_i$$

Modules of $WM(4,5)$

$V' \hookrightarrow H \otimes H$, $WM(4,5) = \frac{V'}{I'}$



Modules of $WM(4,5)$



Future Work

- Look at examples $WM(p,q)$ for higher p,q
- Consider coset construction $WM(3,5) = \frac{\widehat{\mathfrak{sl}}(3)_{-\frac{3}{2}} \otimes \widehat{\mathfrak{sl}}(3)_1}{\widehat{\mathfrak{sl}}(3)_{-\frac{1}{2}}}$

Thank you